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Effects of randomness of grain boundary resistance on fatigue initiation life

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Abstract

In the framework of crack number density analysis, a theoretical investigation is carried out on the relationship between the statistical characteristics of fatigue initiation life and the stochastic growth of short fatigue cracks. The numerical results indicate that, compared with the influence of grain boundary resistance, the effect of the degree of randomness of crystallographic crack growth is only secondary.

Keywords: Randomness; Grain boundary; Short fatigue cracks; Number density; Fatigue initiation life

1. Introduction

For a variety of metallic materials, such as steels and aluminum alloys, fatigue damage is a serous problem limiting the service life of structures. Very often, the service life consists of the fatigue initiation stage, where no long cracks can be detected, and the fatigue growth stage, where one or a few long cracks dominate the material behavior [1–3]. In many cases, especially for high-cycle fatigue damage evolution, the fatigue initiation life takes a major portion of the service life [4,5], and understanding it is immensely important to developing longer fatigue life materials and advanced manufacturing techniques.

One of the intrinsic difficulties in predicting fatigue life is that the experimental results, even for nominally same materials subjected to similar loadings and environments, can vary in large ranges, which was attributed to the residual stresses, geometrical factors, and microstructures [6,7]. Usually, for long cracks the first two factors are pronounced; and for short cracks the microstructure comes in by affecting both crack nucleation and crack growth. In the fatigue initiation stage, the extent of damage is determined by the evolution of collective short fatigue cracks (SFC) [8]. The crack nucleation is usually related to the intrusion and extrusion of persistent slip bands [9,10], and the SFC growth is strongly dependent of the local grain structure [11–13]. In a recent study on fatigue crack growth across high angle grain boundaries in a Fe-2 wt.% Si alloy [14], it was found that the crystallographic crack growth rate, da/dN, is quite random inside grains. The standard deviation could be around 50% of the mean value. On the other hand, the additional resistance offered by grain boundaries resulted in a significant retardation effect. In order to transmit across a grain boundary, the fracture surfaces must be geometrically necessarily branched, with the persistent grain boundary islands between break-through points being separated through shear deformation combined with shear fracture. As a result, the average growth rate associated with the front transmission was lower than the crystallographic growth rate by 4–5 orders of magnitude [14].

In the early stage of fatigue damage evolution, especially in ductile metals, the damage is highly dispersed. There usually do not exist localized damage zones where a few major defects control the material performance. Consequently, the evolution of SFC population must be considered as a whole. In this article, we examine the effects of randomness of SFC growth on the aggregate material response based on a crack number density (CND) analysis.

2. Governing equations

For a system with nearly uniformly dispersed SFC, the extent of damage can be described by the crack number

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density, $n(a,N)$, which is defined as the number of SFC of length a at load cycle N. In order to obtain the $n-a$ distribution curve, we can measure the length of every crack in the material, or, more realistically, solve the equilibrium equation [11]

$$\frac{\partial n(a,N)}{\partial N}+\frac{\partial[A(a,N)\cdot n(a,N)]}{\partial a}=n_N(a,N) \tag{1}$$

where $A(a,N)$ is the effective growth rate of SFC of size a at load cycle N, and $n_N(a,N)$ is the effective crack nucleation rate, that is, the number of SFC of size a nucleated at load cycle N.

The fatigue initiation life, N_{in}, can be defined as the number of load cycles required to form at least one crack that is longer than the critical value, a_{cr}, i.e.

$$n(a_{cr},N_{in})\geq 1 \tag{2}$$

Here, we assume that when $a>a_{cr}$, the barrier effect of grain boundaries is negligible.

As previously discussed, in order to gain a deep insight into the statistical characteristics of the fatigue initiation life, the stochastic nature of the growth of individual SFC must be taken into account. The effective growth rate $A(a,N)$ can be stated as

$$A(a,N)=\bar{A}(a,N)+\delta A(a,N) \tag{3}$$

where $\bar{A}$ is the average growth rate and δA is the stochastic 'noise'.

In a previous study on the SFC growth in a stainless steel [11], a linear simplification was employed to assess $A(a,N)$. In order to take account for the nonlinearity, we assume

$$\bar{A}=\alpha(\Delta\sigma\cdot\sqrt{a})^m\left[\left(\frac{x}{d}\right)^n+A_0\right] \tag{4}$$

where α, m, and n are material constants, $\Delta\sigma$ is the stress amplitude, x is the distance from the crack tip to the next grain boundary, d is the grain size, and A_0 is the average effective growth rate associated with front transmission across grain boundaries. The term in the first bracket is a variant of Paris law, reflecting the crystallographic growth behavior. The term in the second bracket captures the retardation effect of grain boundaries, which is a simplification of the equation developed by Navarro and de los Rios [15]. If there were no grain boundary effect, the first term would dominate and the growth rate would increase with crack length, a. In a polycrystalline material, however, as the crack tip enters the grain boundary affected zone, the growth rate is reduced significantly. In a variety of materials, most of the SFC can never break through any grain boundary and remain grain-sized; even for a SFC that eventually grows into a long crack, it often takes tens of thousand of cycles to overcome a single grain boundary [14]. Therefore, the value of A_0 should be of the order of $10^{-5}-10^{-4}$.

In a field of randomly oriented grains, there is no correlation among the misorientations of grain boundaries encountered by the crack tip. Even inside a grain, because a significant portion of the crack front is trapped by grain boundaries in the interior, the pattern of the variation of da/dN is quite random [14]. Thus, the deviation of crack growth rate from $\bar{A}$ can be stated as

$$\delta A(a,N)=L(a)\cdot W(N) \tag{5}$$

where $W(N)$ is Gauss white noise, and $L(a)$ reflects the crack size effect, which, for the first order approximation, can be relate to $\bar{A}$ via

$$L(a)=\alpha'\bar{A}^{\beta} \tag{6}$$

where α' and β are material parameters. If $\beta=0$, $L(a)$ is a constant and δA is crack size independent; if $\beta>0$,δA increases with $\bar{A}$, as has been observed in experiments [16].

Substituting Eqs. (3) and (5) into (1) gives

$$\frac{\partial n(a,N)}{\partial N}+\frac{\partial[\bar{A}(a,N)\cdot n(a,N)]}{\partial a}=n_N(a,N)-\frac{\partial[L(a)\cdot n(a,N)]}{\partial a}W(N), \tag{7}$$

which can be rewritten in the form of Ito's formula [17]

$$dn_j(N)=H_0+H_1dB_j \tag{8}$$

where $H_0=-(\bar{A}_j/\Delta a)n_j(N)+(\bar{A}_{j-1}/\Delta a)n_{j-1}(N)+n_{Nj}$; $H_1=(L_{j-1}/\Delta a)n_{j-1}(N)-(L_j/\Delta a)n_j(N)$; $j=1, 2, 3\ldots$; Δa is a small crack length increment; the subscript j indicates the value at $a=j\cdot\Delta a$; and B is a Wiener process.

Assume that initially the material is defect free, that is, $n_j(0)=0$. When $\int_0^\infty n(a,N)da$ is relatively large, Eq. (8) has the solution

$$n_j(N)=\left[\exp\left(-\frac{\bar{A}_j}{\Delta a}N\right)\right]\cdot\left\{\int_0^N\left[\exp\left(\frac{\bar{A}_j}{\Delta a}\tilde{N}\right)\cdot\left(\frac{\bar{A}_{j-1}}{\Delta a}n_{j-1}(\tilde{N})+n_{Nj}(\tilde{N})+H_1(\tilde{N})dB_j\right)\right]d\tilde{N}\right\} \tag{9}$$

where '$\int$' denotes mean square integration. The determinant and the random parts of the crack number density can then be obtained as

$$\bar{n}_j(N)=\left[\exp\left(-\frac{\bar{A}_j}{\Delta a}N\right)\right]\cdot\left\{\int_0^N\left[\exp\left(\frac{\bar{A}_j}{\Delta a}\tilde{N}\right)\cdot\left(\frac{\bar{A}_{j-1}}{\Delta a}n_{j-1}(\tilde{N})+n_{Nj}(\tilde{N})\right)\right]d\tilde{N}\right\} \tag{10}$$

and

$$\delta n_j(N)=\left[\exp\left(-\frac{\bar{A}_j}{\Delta a}N\right)\right]\cdot\left\{\int_0^N\left[\exp\left(\frac{\bar{A}_j}{\Delta a}\tilde{N}\right)\cdot H_1(\tilde{N})dB_j\right]d\tilde{N}\right\}, \tag{11}$$

respectively.

According to Eq. (2), at the end of the fatigue initiation stage,

$$n_J(N_{\rm in}) = 1 \tag{12}$$

where $J = a_{\rm cr}/\Delta a$. Thus, the mean value of $N_{\rm in}$, $E_{\rm N}$, can be calculated through

$$E_{\rm N} = f(n_J = 1) \tag{13}$$

where f is the inverse function of Eq. (10); and the standard deviation of $N_{\rm in}$, $\delta_{\rm N}$, is

$$\delta_{\rm N} = \sqrt{\frac{D(\delta n_J)}{(\partial n_J/\partial N)}}, \tag{14}$$

where, based on Eq. (11),

$$D(\delta n_j) = \int_0^N \left\{ \left[\exp\left(-\frac{\bar{A}_j}{\Delta a}(N-\tilde{N}) \right) \right] \cdot \left[-\frac{\partial (L_j n_j)}{\partial a} \right] \right\}^2 {\rm d}\tilde{N} \tag{15}$$

is the variance of δn_j. Through Eq. (9), we have

$$\frac{\partial n_j}{\partial N} = \left[\exp\left(-\frac{\bar{A}_j}{\Delta a} N \right) \right] \cdot \left[\exp\left(\frac{\bar{A}_j}{\Delta a} \tilde{N} \right) \cdot \left(\frac{\bar{A}_{j-1}}{\Delta a} n_{j-1}(\tilde{N}) + n_{Nj}(\tilde{N}) + H_1(\tilde{N}){\rm d}B_j \right) \right] - \frac{\bar{A}_j}{\Delta a} n_j(N) \tag{16}$$

Based on Eqs. (4)–(6) and (13)–(16), the mean value and the standard deviation of fatigue initiation life can be calculated as functions of $\{\alpha, m, n, A_0, \alpha', \beta, a_{\rm cr}/d\}$.

3. Results and discussion

The parameter m is an analog to the Paris law constant, with the theoretical value of 2 [3]. For most of materials, it is in the range of 2–4, and the value of n ranges from 0.5 to 2 [8]. The parameter α resembles the coefficient in the Paris law, and its value for a variety of materials is available in open literature, e.g. [18]. For a stochastic process with negligible self correlation, $\beta = 1$. Thus, the degree of randomness is described by a single parameter α'. If the range of α' is taken as 0–0.5, the ratio of the standard deviation to the mean value of da/dN is about 0–0.3, which seems plausible. In the following discussion, the grain size d is set to 10 μm, and $a_{\rm cr}$ is taken as $3d$ so as to keep the simulation results in consistent with the experimental data [16].

The mechanisms of crack nucleation are closely related to the behaviors of persistent slips bands [9]. In an experimental study on fatigue behavior of single crystals, it was noticed that only the microcracks longer than about 5 μm can eventually grow into long cracks [10]. The form of $n_{\rm N}$, therefore, can be stated as $n_0\delta(a_{\rm in})$, where the initial crack length $a_{\rm in} = 5$ μm and δ is the Dirac delta function.

Figs. 1 and 2 show the influences of the crystallographic crack growth, which is characterized by m and n, on the mean value and the standard deviation of the fatigue initiation life, respectively. The normalization factor N* is somewhat arbitrarily taken as 5×10^5. The mean value is quite insensitive to both n and m over their expected ranges of variation. As n changes from 0.5 to 2, the variation in $E_{\rm N}$ is less than 5%, and the dependence of $E_{\rm N}$ on m is even weaker, which demonstrates clearly that the growth of short fatigue cracks inside grains, either the contribution of the applied stresses or that of the microstructure, has little influence on the initiation life. It is generally accepted that the crystallographic growth of a short fatigue crack is highly dependent on the persistent slip bands and the resolved shear stresses, and thereby is a function of crystallographic orientations. Consequently, the crack growth rates in different grains can be abundantly different. According to Fig. 1, it can be seen that this phenomenon is not a key factor affecting $E_{\rm N}$. Therefore, a first-order approximation, such as Eq. (4), is quite acceptable.

The effects of m and n on the standard deviation of $N_{\rm in}$, however, is more significant. The variation range of $\delta N/E_{\rm N}$ is about 10% as $\{m,n\}$ changes from $\{0.5, 4\}$ to $\{2,2\}$. The value of $\delta N/E_{\rm N}$ increases with n, which can be attributed to that a larger n leads to a lower crack growth rate in the range of $x/d < 1$, and as a result the degree of data scatter of da/dN

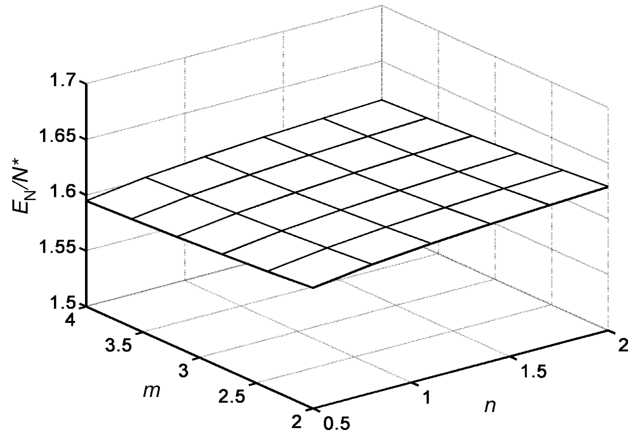

Fig. 1. The mean value of fatigue initiation life, $E_{\rm N}$, as a function of m and n.

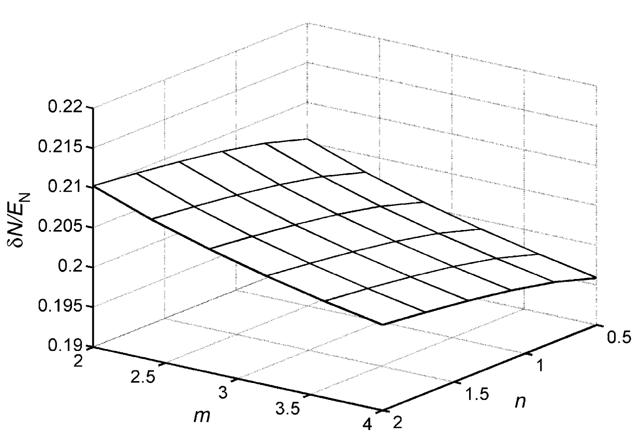

Fig. 2. The standard deviation of fatigue initiation life, $\delta_{\rm N}$, as a function of m and n.

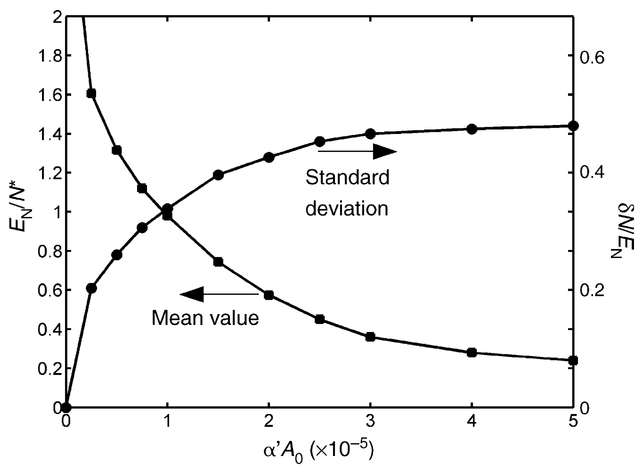

Fig. 3. The effect of $\alpha' A_0$ on E_N and δ_N.

is reduced. For a similar reason, $\delta N/E_N$ is lowered as m rises. Fig. 2 shows that the influence of m is more significant than that of n, that is, the effect of the stress level is more pronounced than that of the microstructure.

The effects of the grain boundary resistance on the fatigue initiation life is shown in Fig. 3, where $m = 1.25$ and $n = 3$. Quite different from the effects of crack growth in the interior of grains, both E_N and $\delta N/E_N$ vary considerably as A_0, which captures the average growth rate of short cracks across grain boundaries, or α', which reflects the stochastic characteristic of grain boundary resistance, changes. When A_0 tends to 0, E_N converges to infinity and δN tends to 0, since the deviation from the mean field behavior vanishes.

When $\alpha' A_0$ is relatively small, E_N and $\delta N/E_N$ are quite sensitive to it. As $\alpha' A_0$ increases, the value of E_N/N^* decreases rapidly, and gradually converges to about 0.2; the value of $\delta N/E_N$ rises rapidly to about 0.43, and, when $\alpha' A_0 > 3$, eventually approaches about 0.44. This is caused by the positive correlation between the crack growth rate and the degree of data scatter. As the grain boundary resistance decreases, the effective crack growth rate becomes larger, and the average fatigue initiation life is lowered. However, the degree of randomness of the fatigue damage evolution is higher. For either E_N or $\delta N/E_N$, the range of variation with the values of $\alpha' A_0$ under consideration is infinity, compared with which, the effect of the crystallographic fatigue crack growth is only secondary.

4. Conclusions

In context of crack number density analysis, the effect of randomness of behavior of individual short cracks on the fatigue initiation life has been discussed in detail. The analysis is not for any specific material. Rather, it is intended for all the ductile materials in which the collective short cracks dominate early evolution of fatigue damage and the grain boundary resistance is significant. The following conclusions are drawn:

1. The SFC growth inside grains has little influence on the mean value of the fatigue initiation life.
2. The value of $\delta N/E_N$ increases as m decrease or n increases, and the influence of the stress amplitude is more important.
3. The factor dominating the fatigue initiation life is the retardation effect of grain boundaries. As A_0 or α' decreases, E_N rises rapidly, especially when the value of $\alpha' A_0$ is relatively low.
4. The standard deviation of fatigue initiation life increases with $\alpha' A_0$.

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