**Strength vs. Toughness**

**Strength**
- Resistance of a material to plastic flow (dislocation motion)

**Toughness**
- Resistance of a material to the propagation of a pre-crack

Two different types of defects, co-existing in all engineering metallic and ceramic materials all the time.

In the framework of linear elastic fracture mechanics (LEFM), we always assume that there are a large number of pre-cracks in any material – similar in spirit with our assumption that crystals always contain dislocations.
Strength vs. Toughness

Strength
If there were no dislocations and dislocation nucleation also did not take place, regular plastic yielding would not happen; the material could be super-strong/hard.

Stiffness
Stiffness of structure
\[ S = \frac{F}{\delta} = \frac{AE}{L_o} \]
\[ S = \frac{F}{\delta} = \frac{C_0 EI}{L^2} \]
……

Young’s modulus
\[ E = \frac{S_b}{a_0} \]
- Bond stiffness
- Bond length at eq’m (atomic size)

Atomic bond

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**Strength**

Maximum loading
- $F_{\text{max}} = \sigma_y A$ (tension/compression)
- $M_{\text{max}} = \sigma_y Z$ (bending)
- $T_{\text{max}} = \tau_{cr} K/R$ (torsion)

Yield strength
- $\sigma_y = 3.1 \tau_y$

Critical resolved shear stress (CRSS)
- $\tau_r = \tau_1 + \tau_2 + \tau_{pe} + \tau_{ub} + \tau_{sh}$

**Fracture**

$K_I = K_{IC}$
- $G = G_c$

Pre-crack propagation
Testing for Toughness

This type of test provides a comparison of the toughness of materials – however, it does not provide a way to express toughness as a material property.

Remote stress applied to a cracked material:

The local stress $\sigma_{\text{local}}$ is proportional to the number of lines of force which rises steeply as the crack tip is approached.

\[
\sigma_{\text{local}} = Y \frac{\sigma \sqrt{\pi c}}{\sqrt{2\pi r}}
\]

* valid when $r \ll c$

- $c$ – crack length
- $r$ – distance from crack tip
- $\sigma$ – remote stress
- $Y$ – geometric constant
Stress Intensity Factor

For any value of $r$, the local stress scales with $\sigma\sqrt{\pi c}$

$$\sigma_{\text{local}} = Y \frac{\sigma\sqrt{\pi c}}{\sqrt{2\pi r}}$$

Mode I stress intensity factor

$$K_1 = Y\sigma\sqrt{\pi c}$$

Mode I indicates tensile loading normal to the crack

- Typically, loading modes are designated by Roman numerals – $K_I$

Critical condition for the crack to advance:

- Rupture of the atomic bond ahead of the crack tip

At $r = r_0$, $\sigma_{\text{local}} = \sigma_c$, when $K_1 = \frac{\sigma_c(2\pi r_0)^{1/2}}{\sqrt{2\pi r}}$

Critical stress intensity factor (fracture toughness)
Stress Intensity Factor

\[ \sigma_{\text{local}} = Y \frac{\sigma \sqrt{\pi c}}{\sqrt{2\pi r}} = \frac{K_I}{\sqrt{2\pi r}} \]

Mode-I stress intensity factor

\[ K_I = Y \sigma \sqrt{\pi c} \]

At critical loading (\( \sigma^* \)) that triggers crack propagation:

\[ \sigma_{\text{local}}(r=r_0) = \sigma_{\text{cr}} \rightarrow \frac{K_I}{\sqrt{2\pi r_0}} = \sigma_{\text{cr}} \rightarrow K_I = \sigma_{\text{cr}} \sqrt{2\pi r_0} = K_{IC} \]

Fracture Toughness

Cracks propagate when the stress intensity factor, \( K_I \), exceeds a critical value – the critical value is known as the fracture toughness \( K_{IC} \).

Different definitions of \( c \) for through crack and edge crack

\[ K_I = Y \sigma \sqrt{\pi c} \]

When \( \sigma = \sigma^* \)

\[ K_I = Y \sigma^* \sqrt{\pi c} \]

- A material parameter: Fracture toughness, critical stress intensity factor
- Unit: MPa\( \sqrt{\text{m}} \)
Example

The fracture toughness of a material is 4 MPa·m^{1/2}. The material will work under a tensile force of 10,000 N, in the form of a round rod with the cross-sectional radius of 10 mm. A 2 mm long edge crack is detected on the rod surface. Is this rod still safe? Assume the geometric factor $Y = 1$.

\[
\sigma = \frac{F}{A} = \frac{10000 \text{ N}}{\pi \times (10 \text{ mm})^2} = 31.8 \text{ MPa}
\]

\[
K_I = Y\sigma(\pi c)^{1/2} = 31.8 \text{ MPa} \times (\pi \times 0.002 \text{ m})^{1/2} = 2.52 \text{ MPa·m}^{1/2}
\]

Because $K_I < K_{IC}$, the crack will not propagate, so the rod is safe.

At the critical crack length, $K_I = K_{IC}$. That is,

\[
Y\sigma(\pi c_{cr})^{1/2} = K_{IC}
\]

\[
31.8 \text{ MPa} \times (\pi \times c_{cr})^{1/2} = 4 \text{ MPa·m}^{1/2}
\]

\[
c_{cr} = 0.005 \text{ m or 5 mm}
\]

As long as the crack is shorter than 5 mm, the rod is safe.

Energy Release Rate $G$ and Fracture Resistance $G_c$

For a crack to grow, sufficient external work must be done which is in the form of released elastic energy.

\[
G = \frac{\partial U}{\partial c} \quad \text{(A loading condition)}
\]

\[
G_c = 2\gamma: \text{critical energy release rate, fracture resistance (A material parameter)}
\]

\[
K_{IC} = \sqrt{EG_c}
\]

$\gamma$ – Effective surface energy

$G_c$ – Required energy to separate a unit area of fracture surfaces

Unit of $G$ and $G_c$: J/m² or N/m
**Example** We have a composite material, but its handbook is incomplete. Can we assess its fracture resistance, $G_c$, and fracture toughness, $K_{IC}$?

The following information is known: The stiffness of the composite = 100 GPa; it takes 1 J to pull a single fiber out of the matrix; there are 10 fibers per cm²; the fiber strength is 500 MPa; the matrix strength is 30 MPa. The matrix is brittle.

$G_c = \frac{1 \text{ J}}{10/\text{cm}^2} = 10^5 \text{ J/m}^2 = 100 \text{ kJ/m}^2$

$K_{IC} = \sqrt{\frac{E G_c}{2}} = \sqrt{\frac{10^{11} \text{ Pa} \times 10^5 \text{ J/m}^2}{2}} = 10^8 \text{ Pa}\cdot\text{m}^{1/2} = 100 \text{ MPa}\cdot\text{m}^{1/2}$

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**Energy Release Rate $G$ and Fracture Resistance $G_c$**

For example, consider a double-cantilever beam (DCB).

Crack growth driving force (energy release rate):

$$G = -\frac{(1/b)(\partial U/\partial c)}{b}$$

where $U = \text{stored elastic energy}$; $c = \text{crack length}$

$$U = 2 U_{\text{beam}} (U_{\text{beam}}: \text{stored energy in each beam})$$

$$U_{\text{beam}} = (1/2)F \cdot \delta$$

(each beam behaves as a “spring”;

$\delta = D/2$)

$$= (1/2) S \cdot \delta^2$$

($S = F/\delta = \text{the stiffness of the beam}$;

$I = bh^3/12 \& L = c$)

$\Rightarrow U = 3Eb h^3 \delta^2 / (12c^3) \Rightarrow G = -\frac{(1/b)(\partial U/\partial c)}{b} = 3Eh^3 \delta^2 / (4c^3)$
Energy Release Rate $G$ and Fracture Resistance $G_c$

[Example] A double-cantilever beam (DCB) is 12 cm wide. Each arm is 5 cm high. It is made of the composite material in the previous example ($E = 100 \text{ GPa}; K_{IC} = 100 \text{ MPa} \cdot \text{m}^{1/2}; G_c = 10^5 \text{ J/m}^2$). The crack length is 30 cm. When the tip is opened by a force ($F$) to $D = 4$ cm, will the crack advance?

$$\delta = D/2 = (4 \text{ cm})/2 = 2 \text{ cm} = 0.02 \text{ m}$$

$$G = 3Eh^3/4c^4 = 3 \times (10^{11} \text{ Pa})(0.05 \text{ m})^3(0.02 \text{ m})^2 / [4 \times (0.3 \text{ m})^4]$$

$$= 5.56 \times 10^4 \text{ N/m} < G_c \Rightarrow \text{the DCB is safe}$$

The critical opening distance $\delta_{cr} = 0.027 \text{ m} = 2.7 \text{ cm}$

Brittle ‘Cleavage’ Fracture

Characteristic of ceramics and glasses

Local stress rises as $1/r^{1/2}$ toward the crack tip – if it exceeds that required to break inter-atomic bonds they separate, giving a cleavage fracture
Tough ‘Ductile’ Fracture

Materials contain inclusion which act as stress concentrations when loaded – the inclusions separate from the matrix causing voids to nucleate and grow, causing fracture.

Ductile Fracture of Cracked Sample

If a material is ductile, a plastic zone forms at the crack tip.

Within the plastic zone, voids nucleate, join, and link to cause fracture.

The plasticity blunts the crack tip, reducing the severity of the stress Concentration.

Crack tip opening displacement: \( \text{CTOD} = \frac{K_t^2}{(\sigma_y E)} \)
Brittle Fracture  Cup-and-Cone Ductile Fracture

Cleavage facet in an individual grain  Dimple (final void)

Cleavage  Fibrous
From fractography, the dimple size in a metallic material is $2r \sim 4 \mu m$. The stiffness and the strength of the material are $E = 70$ GPa and $\sigma_y = 100$ MPa, respectively. Estimate its fracture toughness.

\[ \text{Work to form a single dimple} = \sigma V \]
\[ = (100 \text{ MPa})[(4/3)\pi \times (4 \mu m/2)^3] \]
\[ = 3.35 \times 10^{-9} \text{ J} \]

\[ \text{Number of dimples per unit area} \]
\[ = 1/[(\pi \times (4 \mu m/2)^2)] = 8.0 \times 10^{10}/\text{m}^2 \]

\[ G_c = (3.35 \times 10^{-9} \text{ J}) (8.0 \times 10^{10}/\text{m}^2) \]
\[ = 268 \text{ J}/\text{m}^2 \]

\[ K_c = (E \cdot G_c)^{1/2} = [(70 \times 10^9 \text{ Pa})(268 \text{ J/m}^2)]^{1/2} \]
\[ = 4.3 \times 10^6 \text{ Pa}\cdot\text{m}^{1/2} = 4.3 \text{ MPa}\cdot\text{m}^{1/2} \]

This is merely a very rough estimate for self-comparison purpose!

\[ K_{IC} \sim \sqrt{E G_c} \sim \sqrt{\frac{\sigma_y \left(\frac{8 \pi r^3}{3}\right)}{\pi r^2}} \sim \sqrt{\frac{4}{3} E \sigma_y r} = \xi \sqrt{E \sigma_y r} \]

**Example** From fractography, the dimple size in a metallic material is $2r \sim 4 \mu m$. The stiffness and the strength of the material are $E = 70$ GPa and $\sigma_y = 100$ MPa, respectively. Estimate its fracture toughness.

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**Example** From fractography, the dimple size in a metallic material is $2r \sim 4 \mu m$. The stiffness and the strength of the material are $E = 70$ GPa and $\sigma_y = 100$ MPa, respectively. Estimate its fracture toughness.

**Or, if we assume the largest dimple size is roughly the same as the critical crack opening displacement (CTOD):**

\[ K_{IC} = [\text{CTOD: } \sigma_y E]^{1/2} \]
\[ = [(10 \times 10^4 \text{ m})(100 \times 10^6 \text{ Pa})(70 \times 10^9 \text{ Pa})]^{1/2} \]
\[ = 8.4 \times 10^6 \text{ Pa}\cdot\text{m}^{1/2} = 8.4 \text{ MPa}\cdot\text{m}^{1/2} \]

This is merely a very rough estimate for self-comparison purpose!

\[ A \text{ correction factor } \xi (1/3 \text{ to } 3) \text{ may be employed} \]
Fracture Toughness – Modulus Chart

Values range from 0.01 – 100 MPa.m\(^{1/2}\)

Fracture Toughness – Strength Chart

Values range from 0.01 – 100 MPa.m\(^{1/2}\)
A plastic zone forms at the crack tip where the stress would otherwise exceed the yield strength.

**Size of process zone:**

\[ r_y = 2 \left( \frac{\sigma^2 \pi c}{2 \pi \sigma_y^2} \right) = \frac{K^2_1}{\pi \sigma_y^2} \]

A material transitions from yield to fracture at a critical crack length.

Crack length necessary for fracture at a materials yield strength – Maximum crack length that does **NOT** lead to material weakening:

\[ c_{\text{crit}} = \frac{K^2_1}{\sigma_y^2 \pi \sigma_y^2} \]

Fracture stress: Stress required for fracture for a given crack length:

\[ \sigma_f = \frac{K_{1c}}{\sigma_y \sqrt{\pi c}} \]

Failure strength \((\sigma_{fa}) = \sigma_y \text{ or } \sigma_f, \text{ whichever is lower}\)
Critical crack lengths are a measure of the damage tolerance of a material

<table>
<thead>
<tr>
<th>Material class</th>
<th>Transition crack length, ( c_{\text{tr}} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metals</td>
<td>1–1000</td>
</tr>
<tr>
<td>Polymers</td>
<td>0.1–10</td>
</tr>
<tr>
<td>Ceramics</td>
<td>0.01–0.1</td>
</tr>
<tr>
<td>Composites</td>
<td>0.1–10</td>
</tr>
</tbody>
</table>

Tough metals are able to contain large cracks but still yield in a predictable, ductile, manner

Ductile-to-Brittle Transition

At low temperatures some metals and all polymers become brittle

As temperatures decrease, yield strengths of most materials increase leading to a reduction in the plastic zone size

Metals with an FCC or HCP structure remain ductile at the lowest temperatures
Ductile-to-Brittle Transition

Stronger yet less tough

Brittle Fracture Strength $\sigma_f$, at which $K_I = K_{IC}$ (the precrack length, $c$, may be assumed as the grain size, $d$)

Yield strength, $\sigma_y$

Brittle-to-Ductile Transition Temperature, $T_{BD}$

Stress

Precrack

Temperature

Davidenkov Diagram: Competition between plastic deformation vs. fracture

Embrittlement from Chemical Segregation

Impurities in an alloy are normally found in grain boundaries – this leads to a network of low-toughness paths that can lead to brittle fracture

Hydrogen embrittlement:
- Fractography analysis
- Elemental analysis
- Comparison testing

Reference sample: thermally treated at 150-250 °C
The Strength-Toughness Trade-Off

Increasing the yield strength of a metal decreases the size of the plastic zone surrounding a crack – this leads to decreased toughness.

Manipulating Polymers

Fillers, impact modifiers, and fiber reinforcement can significantly alter the fracture toughness of polymers.
Toughening by Fibers

When a crack grows in a matrix, the fibers remain intact and bridge the crack

A Case Study

Can we (human being) develop “martian bricks” by using raw martian soils?
- This is an urgent request from NASA
  (human exploration mission by 2030’s)
- A real research project began in 2012

To build martian bases and outposts
To maintain martian bases/outposts
To fabricate massive and bulky structural parts in large-scale space telescopes, storage units, waste disposal sites, launch and landing platforms…